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New ideas for a transducer layout in a spherical GW antenna

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We discuss a general procedure to find the coupled motion of a set of resonant transducers attached to a spherical GW detector, and present some preliminary results. A new specific proposal for a polyhedric, quasi-spherical antenna is also presented.

1. INTRODUCTION

It is widely believed within the community of experts in GW detection that future developments in resonant detectors clearly point in the direction of spherically shaped antennæ. The excellence of this shape derives from its omnidirectionality and its large cross section for GW absorption, even at higher resonance modes —see [1] and references therein. The spherical detector is also particularly well adapted to sense *metric* tidal GW excitations, due to the monopole-quadrupole nature of the latter in the general case [2].

In order to take positive advantage of the sphere, however, one has to be able to mount, and set to work, a suitable system of electromechanical transducers, which will amplify the extremely tiny oscillations of the antenna's surface, and convert them to readable output. Currently working bars [3] make use of a *resonant* transducer, attached to one of the bar's end faces, whose frequency is accurately matched to the cylinder's first longitudinal resonance. When it comes to a sphere, however, a number of complications enter the scenario. For example, the quadrupole modes of a sphere are 5-fold degenerate, so a minimum 5 transducers, all tuned to the *same* frequency, are required to sort out the corresponding amplitudes. A multi-transducer layout is naturally more complex than a single transducer one, and so new problems have to be solved.

W. Johnson and S. Merkowitz, from LSU, have pioneered in the recent years an idea for a transducer layout, so called *TIGA* [4], and performed a first round of measurements on a prototype model [5]. Although excellent agreement between theory and experiment is reported by these authors, the presence of minor, fine structure, discrepancies is acknowledged, too.

We have independently developed a theoretical model to assess the motion of the system with a somewhat different philosophy. We expect our model analysis to be sufficiently general to account, amongst other, for the just mentioned fine structure details, but further expansion is still required, as we are at the moment only in the first stages of its development. In what follows, we explain the main ideas underlying our model, as well as some of the preliminary results already found. Particularly interesting amongst these is a new proposal we make for a quasi spherical, polyhedric antenna with remarkably complete potentialities as a GW detector, as it can also be operated as a *multi frequency* device.

2. GENERAL EQUATIONS

We shall be assuming that a solid elastic sphere of mass M and radius R is endowed with a set of N transducers, attached to its surface at locations $\mathbf{x}_A$, $(A=1,\dots,N)$. The transducers will be modelled as identical simple harmonic oscillators, each of mass M_t and resonance frequency Ω. If an external density of force $\mathbf{f}(\mathbf{x},t)$ is acting on the sphere then the equations of motion for the coupled system are [6]

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2}-\mu\nabla^2\mathbf{u}-(\lambda+\mu)\,\nabla(\nabla\cdot\mathbf{u})\;=$$

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$$= \mathbf{f}(\mathbf{x},t) + \eta M\Omega^2 \times$$
$$\times \sum_{A=1}^{N} \delta^{(3)}(\mathbf{x}-\mathbf{x}_A)\,[\xi_A(t)-\mathbf{n}_A\cdot\mathbf{u}(\mathbf{x}_A,t)]\,\mathbf{n}_A \quad (1)$$
$$\ddot{\xi}_A(t) = -\Omega^2\,[\xi_A(t)-\mathbf{n}_A\cdot\mathbf{u}(\mathbf{x}_A,t)] \quad (2)$$

where $\mathbf{u}(\mathbf{x},t)$ is the field of sphere displacements, and $\xi_A(t)$ the displacement of the A-th transducer relative to the undeformed sphere surafce; ρ, λ, μ are the density and elastic Lamé coefficients, respectively, and $\eta \equiv M_t/M$. Although the external force $\mathbf{f}(\mathbf{x},t)$ in (1) can be anything, we shall concentrate here on a GW excitation; for a *general metric* GW the driving term admits the decomposition [2]

$$\mathbf{f}(\mathbf{x},t) = \mathbf{f}^{(S)}(\mathbf{x})\,g^{(S)}(t) + \sum_{m=-2}^{2} \mathbf{f}^{(m)}(\mathbf{x})\,g^{(m)}(t) \quad (3)$$

where $g^{(S)}(t)$ is the *monopole* amplitude of the Riemann tensor, while $g^{(m)}(t)$ are its *quadrupole* amplitudes. $\mathbf{f}^{(S)}(\mathbf{x})$ and $\mathbf{f}^{(m)}(\mathbf{x})$ are tidal form factors.

A formal solution to eqs. (1) can be written down in terms of Green functions [2], but it is so very complicated that symplifying assumptions ought to be made in order to extract valuable information. First of all, the transducer mass is *much smaller* than the sphere's. It is therefore advantageous to find a solution as a *perturbative expansion* in η, then retain a suitable number of terms in it for each specific purpose. Also, we shall be assuming that the transducer frequency Ω is equal to one of the sphere's resonances ω_{nl}, normally a quadrupole or a monopole harmonic. It turns out after very lengthy algebra that, to lowest order in η, there is a linear relationship between the driving terms $\hat{g}^{(\alpha)}(s)$, $\alpha = S, m$ ($m = -2,...,2$), and the system's response $\hat{y}_A(s)$ given by the following equations:

$$\hat{y}_A(s) = \eta^{-1/2} \sum_{\alpha} \Lambda_A^{(\alpha)}(s;n,l)\,\hat{g}^{(\alpha)}(s) \quad (4)$$

where $y_A(t) \equiv \xi_A(t) - \mathbf{n}_A\cdot\mathbf{u}(\mathbf{x}_A,t)$. These equations are written in terms of Laplace transforms, noted by circumflex accents on symbols, for expediency. $\Lambda_A^{(\alpha)}(s;n,l)$ is therefore a *transfer function* matrix. The first thing which is readily visible in (4) is that there is a *mechanical* enhancement factor $\eta^{-1/2}$ in the system response with respect to the GW amplitudes $\hat{g}^{(\alpha)}(s)$, exactly the same as happens with bars. A deeper scrutiny of the equations requires precise specification of the matrix elements $\Lambda_A^{(\alpha)}(s;n,l)$, of course. Lack of space in this short communication dictates that only a qualitative description be presented here.

The information contained in $\Lambda_A^{(\alpha)}(s;n,l)$ is this: i) its *poles*, relative to the Laplace variable s, determine the *characteristic frequencies* of the system, and ii) its specific form, for given n, l, determines the coupling to GWs; it is therefore a *mode pattern matrix*. We find the following, respectively:

i) The system's characteristic frequencies are, for a general transducer distribution, a set of N doublets, practically symmetric around the chosen sphere's frequency ω_{nl}, given by

$$\omega_{A\pm} = \omega_{nl}\left(1 \pm b_A\,\eta^{1/2}\right) \quad (5)$$

where b_A are the eigenvalues of a certain matrix depending only on the positions of the transducers. Again, a remarkable parallelism is found with the situation in bars: the splitting is symmetric around the solid's resonance, and its relative magnitude is proportional to $\sqrt{\eta}$.

ii) The most important feature revealed by the mode pattern matrix is this: if we implant transducers which for example resonate at the first sphere *quadrupole* frequency, ω_{12}, then we can only sense the quadrupole components of the driving force, $\hat{g}^{(m)}(s)$; the system will be (practically) insensitive to the monopole component $\hat{g}^{(S)}(s)$, even if it is large at the observation frequency. Likewise, if $\Omega = \omega_{10}$, the first *monopole* frequency, then the system will not sense the quadrupole amplitudes $\hat{g}^{(m)}(s)$, even if they are present in the signal at that frequency.

These facts have suggested us that a suitable distribution of transducers on the sphere's surface may be advantageously used to build a rather complete GW antenna. We describe our proposal in the next section.

3. A NEW PROPOSAL

As just described, a set of resonant transducers is only sensitive to the amplitudes of that particular mode of the sphere to whose frequency the transducers are tuned. Interferences between modes are weak, if they are sufficiently separated in frequency. It is therefore very attractive to make a design whereby *several* modes of the sphere can be sensed in parallel. These could be the *first and second quadrupole*, and the *first monopole*. The frequencies of these modes are in the ratios 1/1.92/2.05, respectively, so quite clearly separated.

The choice is not gratuitous: the *second* quadrupole cross section for GW energy absorption is still large —cf. [1]—, while, on the other hand, almost every theory of the gravitational field, other than General Relativity, predicts the existence of scalar radiation, thence the use of a monopole mode sensor should contribute valuable experimental evidence for or against the hypothesis of such kind of radiation.

The next question is naturally *where* to place so many transducers: 11 would be needed, as there are two quadrupole, five-fold degenerate, modes, and one monopole, non-degenerate, mode in the above proposal. We have considered substituting the sphere by a suitable *polyhedron*, following the philosophy of Johnson and Merkowitz of ease of mounting and manipulation. After looking at a number of different alternatives, we have found a very appealing one in the so called *pentagonal hexacontahedron*. This is a *sixty* face convex body, whose faces are all *identical*, though irregular, pentagons. It has six axes of quintuple symmetry, like the icoshedron, but it is much more close to the sphere in volume and area. Finally, a sphere can be *inscribed* in this polyhedron, which is tangent to all faces at a certain *centre of the face*. If transducers are attached to such centres, they will all be equidistant from the centre of the sphere —a very good simulation of the theoretical sphere.

In Figure 1, we give a schematic graphical representation of our proposed device, see the caption for details. The concrete choice for the positions has been established in terms of the best possible *equanimity* of sensitivity to all five quadrupole amplitudes $\hat{g}^{(m)}(s)$, i.e., in such a way that signal-to-noise ratios for all these modes are as close as possible to one another. Details of this and other questions will be given shortly in a journal paper.

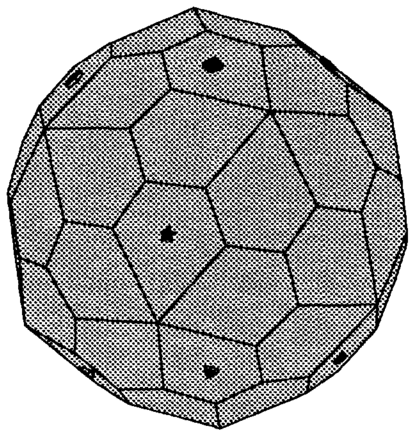

Figure 1. The proposed polyhedric antenna. Transducers are marked as follows: a *square* for the first quadrupole frequency, a *triangle* for the second, and a *star* for the monopole.

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